#2.

$p(1) = 0$ $\quad c\,p(x)$ $\quad c\,p(1) = 0$

$q(1) = 0$ $\quad p(x) + q(x)$

$p(1) + q(1) = 0$

$\int_0^5 c\,p(t)\,dt = 0 \qquad \int_0^5 q(t)\,dt = 0$

$p(t) + q(t)$? $\quad \int_0^5 (p(t) + q(t))\,dt = \int_0^5 p(t)\,dt + \int_0^5 q(t)\,dt$

$= 0 + 0 = 0$

$f'(8) = f(6)$, $\quad g'(8) = g(6)$

$(f+g)'$? $\quad (f+g)'(8) = f'(8) + g'(8)$

$\qquad\qquad\qquad\qquad = f(6) + g(6) = (f+g)(6)$

#3.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Kernel = null space.

#6.

The set

$$B = \left\{ \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} \right\}$$

is a basis of the space of upper-triangular 2×2 matrices.

Find the coordinates of $M = \begin{bmatrix} 5 & -9 \\ 0 & -3 \end{bmatrix}$ with respect to this basis.

$$\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

$A^{-1}Ax = A^{-1}b$

$x = A^{-1}b$

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$B \longrightarrow M_B \longrightarrow M_B^{-1}$

$\longrightarrow M_B^{-1} \cdot v$

$$c_1\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + c_2\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} + c_3\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 5 \\ -9 \\ -3 \end{bmatrix}$$

M_B^{-1}

$$M_B^{-1}\underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -2 \end{bmatrix}}_{M_B}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = M_B^{-1}\begin{bmatrix} 5 \\ -9 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \boxed{M_B^{-1}\begin{bmatrix} 5 \\ -9 \\ -3 \end{bmatrix}}$$

$C^T = \mathrm{adj}(A)$

$$\boxed{\frac{1}{\det(A)} C^T} A = \cancel{\det(A)} I$$

$$\| \quad A^{-1}$$

$$C^T = \det(A) A^{-1} \Rightarrow C = \underline{\det(A) (A^{-1})^T}$$

#9. Determine the values of k so that the vectors $\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} k & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 0 & 3 \end{bmatrix}$ are linearly dependent. If your answer is a fraction, enter it as a fraction.

$k =$

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} 2 & k & 2 \\ 3 & 1 & -2 \\ -1 & 1 & 3 \end{bmatrix}\right) = 6 + 2k + 6 + 2 + 4 - 9k$$

$$= -7k + 18 = 0 \qquad 7k = 18 \qquad k = 18/7$$

webwork #7.

The set $B = \{1 - 4x^2,\ 2 - x - 8x^2,\ 4 - 3x - 12x^2\}$ is a basis for P_2. Find the coordinates of $p(x) = 13x - 15 + 44x^2$ relative to this basis:

$\{1, x, x^2\}$

$$\underbrace{\begin{bmatrix} 1 & 2 & 4 \\ 0 & -1 & -3 \\ -4 & -8 & -12 \end{bmatrix}^{-1}}_{M_B} \cdot \begin{bmatrix} -15 \\ 13 \\ 44 \end{bmatrix}$$

#19. B, C are given. base change matrix from basis B to basis C.

$$[x]_C = \underbrace{\boxed{M_C^{-1} M_B}}_{P} [x]_B$$

Webwork. #15

$A \longrightarrow$ reduced echelon form. $\qquad A\vec{x} = \vec{0}$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ \end{array}$$
$$\begin{bmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

basis

$$x_1 = -x_3 + 2x_4$$
$$x_2 = 0 \cdot x_3 - x_4$$
$$x_3 = 1 \cdot x_3 + 0 \cdot x_4$$
$$x_4 = 0 \cdot x_3 + 1 \cdot x_4$$

nullity$(A) = 2$