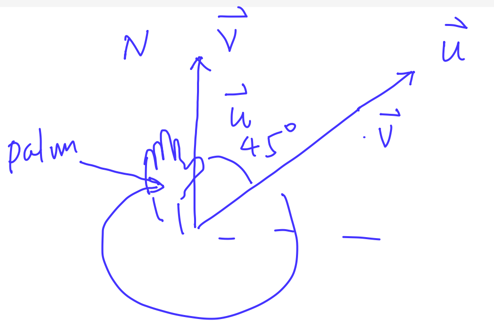
#24

You are looking down at a map. A vector $\mathbf{u}$ with $|\mathbf{u}| = 7$ points north and a vector $\mathbf{v}$ with $|\mathbf{v}| = 10$ points northeast. The crossproduct $\mathbf{u} \times \mathbf{v}$ points:

A) south
B) northwest
C) up
D) down

Please enter the letter of the correct answer: ☐

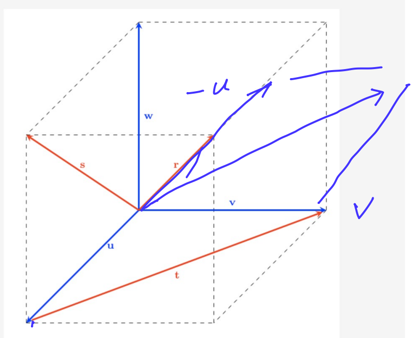
The magnitude $|\mathbf{u} \times \mathbf{v}| =$ ☐



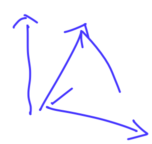

$$|\vec{u} \times \vec{v}| = \|\vec{u}\| \cdot \|\vec{v}\| \sin\theta$$
$$= 7 \cdot 10 \cdot \frac{\sqrt{2}}{2}$$
$$= 35\sqrt{2}$$

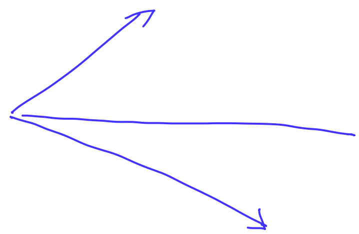
Below is a diagram of the dotted cube with edges of length 2. Suppose vectors $\mathbf{u} = \langle 2,0,0 \rangle$, $\mathbf{v} = \langle 0,2,0 \rangle$, and $\mathbf{w} = \langle 0,0,2 \rangle$.

#5.




$$\langle 2,0,0\rangle + \langle 0,2,0\rangle + \langle 0,0,4\rangle$$
$$= \langle 2,2,4\rangle$$





1. $\mathbf{u} + \mathbf{v} + 2\mathbf{w} =$ ☐

2. $\mathbf{t} =$ ☐ $\quad v - u = \langle 0,2,0\rangle - \langle 2,0,0\rangle = \langle -2,2,0\rangle$

3. $\mathbf{t} - 2\mathbf{r} =$ ☐

4. $\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{r} + \mathbf{s} + \mathbf{t} =$ ☐

#10.

Suppose $\overline{u} = 4\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$, $\overline{v} = 3\boldsymbol{i} - 2\boldsymbol{k}$ and $\overline{w} = 4\boldsymbol{i} - 3\boldsymbol{j} - 2\boldsymbol{k}$.
Compute the following values:

$$|\vec{u}| = \sqrt{16+4+1} = \sqrt{21}$$
$$|\vec{v}| = \sqrt{9+4} = \sqrt{13}$$

$|\overline{u}| + |\overline{v}| =$ ☐ $\quad \sqrt{21} + \sqrt{13}$

$|-5\overline{u}| + 6|\overline{v}| =$ ☐ $\quad 5\sqrt{21} + 6\sqrt{13}$

$|5\overline{u} - 5\overline{v} + \overline{w}| =$ ☐

$\frac{1}{|\overline{w}|}\overline{w} =$ ☐

$\left|\frac{1}{|\overline{w}|}\overline{w}\right| =$ $\boxed{1}$

$$|-5\vec{u}| = 5|\vec{u}| = 5\sqrt{21}$$
$$6|\vec{v}| \qquad 6\sqrt{13}$$

$$\frac{1}{|\vec{w}|} \cdot |\vec{w}|$$

$$|-5\vec{u}| = \sqrt{20^2 + 10^2 + 5^2}$$
$$= \sqrt{5^2 \cdot (21)} = 5\sqrt{21}$$

$$-5\vec{u} = -20\,i - 10\,j - 5\,k$$

## Lecture Notes for 11/30/2022

Review Session for Test 3 which covers 6.1–6.4, 7.1–7.3, 8.1–8.4.

**6.1 Linear transformations between Euclidean spaces**

**6.2 General linear transformations**

**6.3 Isomorphisms**

**6.4 Rank and nullity of a linear transformation**

**7.1 Eigenvalues and eigenvectors**

**7.2 Eigen spaces**

**7.3 Similarity and diagonalization**

**8.1 Introduction to vectors**

**8.2 Vector operations**

**8.3 Dot product**

**8.4 Cross product**

$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 - 5x_3 \\ 3x_1 + 2x_3 \end{bmatrix} \quad // \quad A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad A = \begin{bmatrix} T(e_1), & T(e_2), & T(e_3) \end{bmatrix}$$

$$\begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix}$$

$$= A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 - 5x_3 + 4 \\ x_1^2 \end{bmatrix}$$

$$\{b_1, b_2, b_3\}$$

$$\begin{bmatrix} T(b_1), & T(b_2), & T(b_3) \end{bmatrix}$$

**Chapter 6:**

• Given a transformation, can you tell whether it is a linear transformation?

• If a given transformation is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$, can you write down the matrix of the transformation?

• If a transformation is from a general vector space to another, can you write down the matrix of the transformation with respect to the coordinate vectors under the given bases?

• Can you apply the properties of a linear transformation? (what are the conditions that define a linear transformation?)

• Can you find a basis for the null space of a linear transformation? A basis for the image space (the range) of a linear transformation?

• Can you find the rank and nullity of a linear transformation?

• Can you use the rank and nullity of a linear transformation to determine whether it is 1 to 1, onto, or an isomorphism?

**Chapter 7:**

• The definition of eigenvalue and eigenvector, eigen pair etc;

• Do you know how to compute the characteristic polynomial of a matrix?

• Do you know how to find the eigenvalues of a matrix from its characteristic equation?

• Do you know the definition of an eigenspace corresponding to an eigenvalue and how to find a basis for the eigenspace?

• Do you know the definition of algebraic multiplicity and geometric multiplicity of an eigenvalue and how to determine them? For a special matrix such as a triangular matrix, can you do this quickly?

• Do you know that if we accept complex numbers as eigenvalues and count an eigenvalue with multiplicity $k$ as $k$ eigenvalues, then a matrix $A$ (with real number entries) always has exactly $n$ eigenvalues? Furthermore, the product of these $n$ eigenvalues equals the determinant of $A$?

• Do you know what is the meaning of two matrices being similar to each other?

$B = P^{-1} A P$ $\qquad$ $\underline{M^{-1} A} M = D$

• Do you know what is the meaning of diagonalization of a matrix and what is the condition under which a matrix can be diagonalized?

• Do you know how to find the matrix $M$ so that $M^{-1}AM$ is a diagonal matrix and what is the corresponding diagonal matrix? (Given that we know $A$ can be diagonalized and we also know the eigenvalues of $A$ and a basis for each eigenspace.)

**Chapter 8:**

• The different notations used for vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$;

• Do you know what is the magnitude of a vector? what is a unit vector. Given a vector $\mathbf{v}$, do you know how to find a unit vector that has the same direction as $\mathbf{v}$ or has an opposite direction as $\mathbf{v}$?

• Do you know the geometric meaning of scalar multiplication and addition of vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$? Can you use this knowledge to find the total length of a triangle if the coordinates of the triangles are given? or the length of the diagonals of a parallelogram if two adjacent sides of the parallelogram are defined by two vectors?

• Do you know the definition (and the calculation) of the dot product of two vectors?

• Do you know the geometric meaning of the dot product and be able to use it to determine whether two vectors are perpendicular to each other, or to find the angle between two vectors?

• Do you know the definition of the cross product of two vectors (as given in the determinant form in last class)? Can you calculate the cross product of two vectors?

• Do you know the geometric meaning of the cross product and be able to use it to find the area of the parallelogram with the two vectors as two sides and a vector that is perpendicular to the plane containing the two vectors?

Enter A for your attendance today.

Thank you!

For review purposes, you may want to access the tests and the past webwork assignments.

• I have made your Test 1 and Test 2 accessible to you until December 11 (the day before the final exam). You can review them to re-familiarize yourself about the formats of the tests.

• I will also keep your Test 3 accessible to you (after you have taken it) until December 11. These should help you preparing for the final exam.