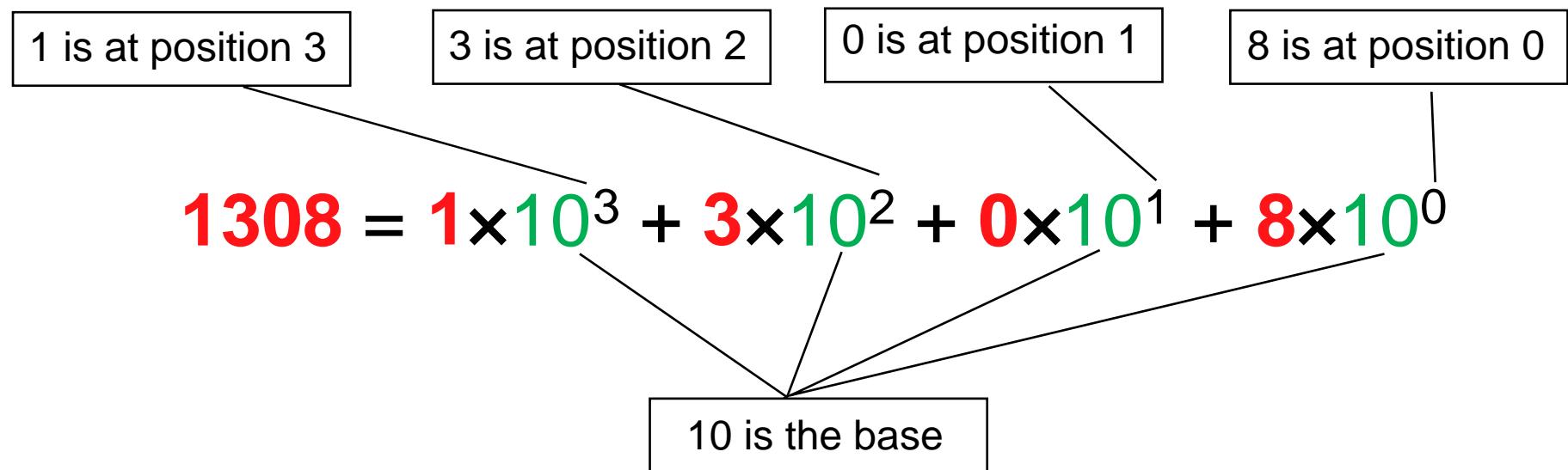


Number Systems I

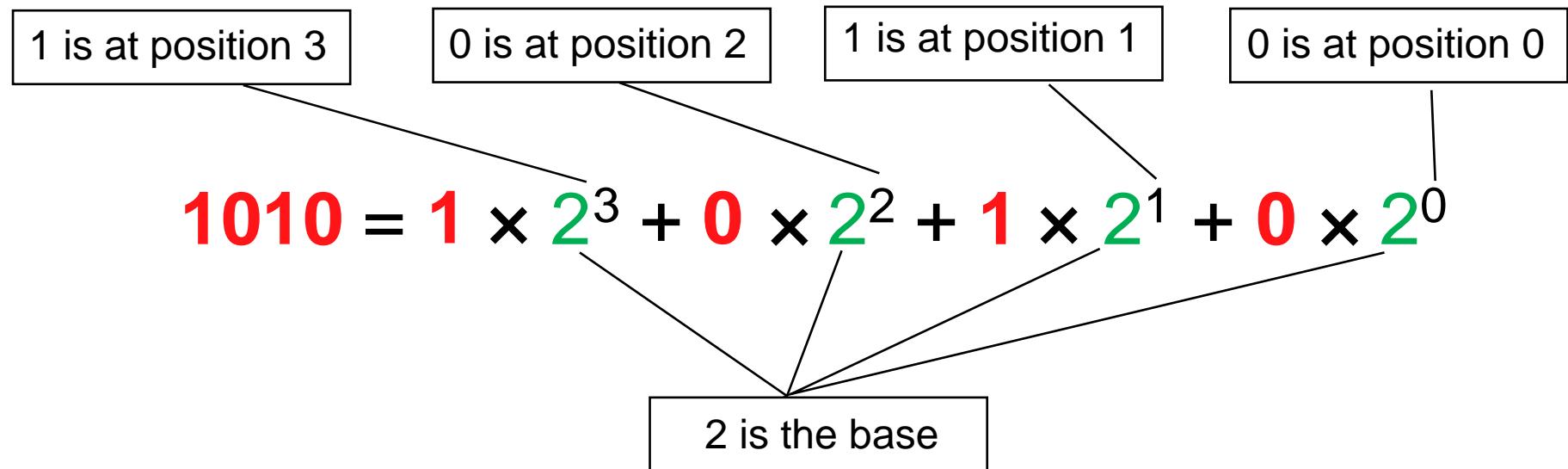
Number Representation

- Decimal Notation: 10 digits (0-9)
- Binary Notation: 2 digits (0,1)
- 10 is the base for Decimal, 2 is the base for binary notation



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Number Representation

- Representation of n base b is called the base b expansion of n

For an integer $b > 1$. Every positive integer n can be expressed uniquely as:

$$n = a_k \cdot b^k + a_{k-1} \cdot b^{k-1} + \cdots + a_1 \cdot b^1 + a_0 \cdot b^0,$$

where k is a non-negative integer, each a_i is an integer in the range from 0 to $b - 1$, and $a_k \neq 0$.

$$1308 = 1 \times 10^3 + 3 \times 10^2 + 0 \times 10^1 + 8 \times 10^0$$

Converting from base b to decimal

For an integer $b > 1$. Every positive integer n can be expressed uniquely as:

$$n = a_k \cdot b^k + a_{k-1} \cdot b^{k-1} + \cdots + a_1 \cdot b^1 + a_0 \cdot b^0,$$

where k is a non-negative integer, each a_i is an integer in the range from 0 to $b - 1$, and $a_k \neq 0$.

$$\begin{aligned}(2012)_3 &= 2 \times 3^3 + 0 \times 3^2 + 1 \times 3^1 + 2 \times 3^0 \\&= 2 \times 27 + 0 \times 9 + 1 \times 3 + 2 \times 1 \\&= 59\end{aligned}$$

Hexadecimal Numbers

- 16 digits
- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111

- A 4-bit binary number can be represented with 1 Hexadecimal Digit
- 1 byte is equivalent to 8 bits
- So, 1 byte can be represented by 2 Hexadecimal digits

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Hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111

$$\begin{aligned}(3B2)_{16} &= 3 \times 16^2 + B \times 16^1 + 2 \times 16^0 \\&= 3 \times 256 + 11 \times 16 + 2 \times 1 \\&= 946\end{aligned}$$

Converting from decimal to base b

For an integer $b > 1$. Every positive integer n can be expressed uniquely as:

$$n = a_k \cdot b^k + a_{k-1} \cdot b^{k-1} + \cdots + a_1 \cdot b^1 + a_0 \cdot b^0,$$

where k is a non-negative integer, each a_i is an integer in the range from 0 to $b - 1$, and $a_k \neq 0$.

$$\left[\begin{array}{l} \text{Base } b \text{ expansion of } (n \text{ div } b) \\ \hline \text{n mod } b \end{array} \right]$$

Converting from decimal to base b

Given $n = 1161$

$$1161 = (3246)_7$$

Find base 7 digits:

$$3 \cdot 7^3 + 2 \cdot 7^2 + 4 \cdot 7 + 6 = 1161$$

Digit in the range
0, 1, ..., 6

Multiple of 7

$$3 \cdot 7^3 + 2 \cdot 7^2 + 4 \cdot 7 + 6 = 1161$$

Rightmost digit is
 $1161 \bmod 7 = 6$

For remaining digits use
 $1161 \div 7 = 165$

$$3 \cdot 7^2 + 2 \cdot 7 + 4$$

$$6$$

For remaining digits use
 $165 \div 7 = 23$

$$3$$

$$4$$

Next digit is
 $165 \bmod 7 = 4$

For remaining digits use
 $23 \div 7 = 3$

$$2$$

Next digit is
 $23 \bmod 7 = 2$

Since 3 is in the range 0, 1, ..., 6

→ All Done!

$$1161 = (3246)_7$$

Example

- Convert the decimal number 542 to a base 5 number.

Number of digits to represent a number

- Given an integer n , the number of digits required to represent n in base b is:

$$\lceil \log_b(n+1) \rceil$$

- Example: how many digits are required to express the base 2 expansion of 13?

$$\lceil \log_2(13+1) \rceil = \lceil \log_2(14) \rceil = \lceil 3.8 \rceil = 4$$