

# Number Systems I

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# Number Representation

- Decimal Notation: 10 digits (0-9)
- Binary Notation: 2 digits (0,1)
- 10 is the base for Decimal, 2 is the base for binary notation

The diagram illustrates the positional notation for the decimal number 1308. It shows the number expanded as a sum of each digit multiplied by a power of 10. The digits are color-coded: 1 (red), 3 (red), 0 (red), and 8 (red). The powers of 10 are color-coded: 10<sup>3</sup> (green), 10<sup>2</sup> (green), 10<sup>1</sup> (green), and 10<sup>0</sup> (green). Four boxes at the top indicate the position of each digit: '1 is at position 3', '3 is at position 2', '0 is at position 1', and '8 is at position 0'. A box at the bottom states '10 is the base'. Lines connect the boxes to the corresponding terms in the equation.

$$1308 = 1 \times 10^3 + 3 \times 10^2 + 0 \times 10^1 + 8 \times 10^0$$

10 is the base

# Number Representation

- Decimal Notation: 10 digits (0-9)
- Binary Notation: 2 digits (0,1)
- 10 is the base for Decimal, 2 is the base for binary notation

The diagram illustrates the positional expansion of the binary number 1010. At the top, four boxes identify the digits and their positions: '1 is at position 3', '0 is at position 2', '1 is at position 1', and '0 is at position 0'. Below these, the equation  $1010 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$  is shown. The digits 1, 0, 1, 0 are in red, and the powers of 2 (2<sup>3</sup>, 2<sup>2</sup>, 2<sup>1</sup>, 2<sup>0</sup>) are in green. Lines connect the boxes to the corresponding terms in the equation. A central box at the bottom states '2 is the base', with lines pointing to each of the four powers of 2 in the equation.

1 is at position 3      0 is at position 2      1 is at position 1      0 is at position 0

$$1010 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

2 is the base

# Number Representation

- Representation of  $n$  base  $b$  is called the base  $b$  expansion of  $n$

For an integer  $b > 1$ . Every positive integer  $n$  can be expressed uniquely as:

$$n = a_k \cdot b^k + a_{k-1} \cdot b^{k-1} + \cdots + a_1 \cdot b^1 + a_0 \cdot b^0,$$

where  $k$  is a non-negative integer, each  $a_i$  is an integer in the range from 0 to  $b - 1$ , and  $a_k \neq 0$ .

$$1308 = 1 \times 10^3 + 3 \times 10^2 + 0 \times 10^1 + 8 \times 10^0$$

# Converting from base b to decimal

For an integer  $b > 1$ . Every positive integer  $n$  can be expressed uniquely as:

$$n = a_k \cdot b^k + a_{k-1} \cdot b^{k-1} + \cdots + a_1 \cdot b^1 + a_0 \cdot b^0,$$

where  $k$  is a non-negative integer, each  $a_i$  is an integer in the range from 0 to  $b - 1$ , and  $a_k \neq 0$ .

$$\begin{aligned} (2012)_3 &= 2 \times 3^3 + 0 \times 3^2 + 1 \times 3^1 + 2 \times 3^0 \\ &= 2 \times 27 + 0 \times 9 + 1 \times 3 + 2 \times 1 \\ &= 59 \end{aligned}$$

# Hexadecimal Numbers

- 16 digits
- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111

- A 4-bit binary number can be represented with 1 Hexadecimal Digit
- 1 byte is equivalent to 8 bits
- So, 1 byte can be represented by 2 Hexadecimal digits

# Hexadecimal Numbers

- 16 digits
- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111

$$\begin{aligned}(3B2)_{16} &= 3 \times 16^2 + B \times 16^1 + 2 \times 16^0 \\ &= 3 \times 256 + 11 \times 16 + 2 \times 1 \\ &= 946\end{aligned}$$

# Converting from decimal to base b

For an integer  $b > 1$ . Every positive integer  $n$  can be expressed uniquely as:

$$n = a_k \cdot b^k + a_{k-1} \cdot b^{k-1} + \cdots + a_1 \cdot b^1 + a_0 \cdot b^0,$$

where  $k$  is a non-negative integer, each  $a_i$  is an integer in the range from 0 to  $b - 1$ , and  $a_k \neq 0$ .

$$\left[ \text{Base } b \text{ expansion of } (n \text{ div } b) \right] \left[ n \bmod b \right]$$



# Converting from decimal to base b

Given  $n = 1161$

$$1161 = (3246)_7$$

Find base 7 digits:

$$3 \cdot 7^3 + 2 \cdot 7^2 + 4 \cdot 7 + 6 = 1161$$

Digit in the range  
0, 1, ..., 6

Multiple of 7

$$3 \cdot 7^3 + 2 \cdot 7^2 + 4 \cdot 7 + 6 = 1161$$

Rightmost digit is  
 $1161 \bmod 7 = 6$

For remaining digits use  
 $1161 \div 7 = 165$

$$3 \cdot 7^2 + 2 \cdot 7 + 4$$

6

For remaining digits use  
 $165 \div 7 = 23$

$$3 \cdot 7 + 2$$

4

Next digit is  
 $165 \bmod 7 = 4$

For remaining digits use  
 $23 \div 7 = 3$

3

2

Next digit is  
 $23 \bmod 7 = 2$

Since 3 is in the range 0, 1, ..., 6  $\longrightarrow$  All Done!

$$1161 = (3246)_7$$

# Example

- Convert the decimal number 542 to a base 5 number.

# Number of digits to represent a number

- Given an integer  $n$ , the number of digits required to represent  $n$  in base  $b$  is:

$$\lceil \log_b (n+1) \rceil$$

- Example: how many digits are required to express the base 2 expansion of 13?

$$\lceil \log_2 (13+1) \rceil = \lceil \log_2 (14) \rceil = \lceil 3.8 \rceil = 4$$